Modern Physics Letters A, Vol. 10, No. 8 (1995) 657–666

GRAVITATIONAL SELF–ENERGY AS THE LITMUS OF REALITY

K. R. W. Jones
Physics Department, University of Queensland, St Lucia 4072, Brisbane, Queensland, Australia

Received 18-Oct-1994
Revised 30-Jan-1995

ABSTRACT

It is argued that the correct physical treatment of self–energy in Newtonian quantum gravity offers a constrained and predictive discriminator for the interpretation of ψ, and thus a clear point of departure for the unification of modern physics.

1. Introduction

Among all, two views on quantum physics have the merit of simplicity. Heisenberg believed that operator non–commutativity was the key, and that the seat of all reality lay with the observer[1]. Schrödinger believed that complex–valued fields distinguished the quantum from the classical, and that the task of physics should be to explain the observed stochasticity of nature in these terms alone[2,3,4,5].

However, at the time, none could say who was right. Thus both views have merged into one indistinguishable blend of the merits in each. In the Copenhagen interpretation[1,6,7] we adopt the stance of Heisenberg upon the observer[1], and the physical tool of Schrödinger in the wavefunction[2]. Operators and wavefunctions are treated upon equal footing[3]. To meld both, in seamless fashion, Dirac has abstracted away those obvious points of difference. In his transformation theory[6] there is no possibility of empirical discrimination — *by definition.*

Is that *really* the case? Indeed, within the confines of the present linear theory it would appear so. However, nature may well find disfavour with the commandments of our theorizing about her. Suppose she prefers to be *nonlinear*[8,9,10,11,14,15,16]. How are we to tell? Can the ultimate philosophical question be made physical[10]?

My thesis here is that the treatment of gravitational self–energy offers a means to discriminate the issue of how ψ should be interpreted. According to the views of Heisenberg we should attempt the construction of a second–quantized theory of gravity, which must have a first–quantized linear wave–equation as its Newtonian limit. In contrast, the historical problem of macroscopic dispersion, for which the Copenhagen observer is traditionally invoked, is here met via a nonlinear treatment of gravitational self–energy appropriate to the Schrödinger interpretation[2]. Thus we identify *two* possible Newtonian theories of quantum gravity, which implies two *entire classes* of relativistic theory, here distinguished via their limits.

2. The incompleteness of Copenhagen quantum physics

The logical self–consistency of quantum physics has long rested upon an unproven assumption. According to Bohr[7], a physical domain exists that is "describable in classical terms". That is in accord with commonsense — but does our theory allow for its construction? The key issue is whether quantum theory exhibits a classical level that is *contained within it, and consistent with its own principles*[11,14,15].

Recently, Jones[15] has demonstrated that the recovery of an exact classical limit is *inconsistent* with the uncertainty principle and must be rejected. Further, a key property assumed in many contemporary treatments of measurement, namely the existence of non–entangling environmental noise[18], is inconsistent with the quantum entanglement of a *linear* theory[4], although *nonlinear* theories permit it[15].

We believe this signals a *genuine problem of logical incompleteness.* To overcome it we seek a *fundamental, physically motivated, and predictive interaction* that is nonlinear, and thus non–entangling[15]. This we seek to invoke in the participatory but separated role of the traditional Copenhagen observer[1,6,7].

3. The problem of the cosmic observer

Consider a free particle. Dispersion spreads the wave–packet[5]. Already, for the simple problem of a universe containing only one particle we require something to arrest this process. In Copenhagen physics an observer finds the particle here or there[7]. The result is localization within the volume of space represented by the accuracy of his measuring device, with the final ψ assigned upon this basis[1].

This process iterates. Should the observer become part of quantum physics, he too is found dispersed — and worse, he is entangled[4]. A new meta–observer must take his place to do this finding. How are we to avoid this fiendish regress of observers observing[17]? Each must sit in the waiting–room of reality until he the non–existent last — he called Godot — arrives. If we are not to conjure a cosmic observer[a] we must find some physics in place of them. Decoherence is very promising[18], but there is not yet a clear physical mechanism to act upon the remnant diagonal terms in a density matrix and so *reduce them.* With a non–entangling field[15] these might be cross–coupled, in noisy fashion, to single out one of them.

If we are to do away with observers, then we must replace them with a *physics of observation.* The strongest demand possible is that the *one–particle* theory be free of an observer. Then self–consistency will not require complexity, although new effects may arise once more particles are included. Fix, therefore, upon the free particle. Look to banish the observer at this the simplest possible level, and to do so *without cost of entanglement*[4]. The problem is then physical; it is dispersion.

[a]Heisenberg has summed up, perhaps obliquely, the cosmical conundrum posed by observers in the following line: "The chain of cause and effect could be quantitatively verified only if the whole universe were considered as a single system — but then physics has vanished, and only a mathematical scheme remains." (consult Ref. 1, p.58). Our aim is to ensure that physics does not so vanish in a quantum theory of cosmic scope, while ensuring, with Einstein (see the article by Pais, Ref. 13, p.899), that God is not rendered a gambler in the human imagination. I find the corollary, *God the observer*, to be an embarassing adjunct to physics, and prefer to avoid it.

4. Towards the unification of micro and macro physics

To locate a *predictive* theory we must confine attention to the known interactions. Certain of these, particularly electromagnetism, are very well–tested and known to introduce quantum correlations. Holding fast to simplicity many options are discounted. To acquire agreement with experiment any change must necessarily be small for an isolated subatomic particle[19]. Dispersion is well–tested in the quantum physics of these[20]. However, it drops rapidly in strength as we pass up towards the many–particle domain of quasi–classical physics. Stimulated by the previous speculations of Kibble[9], Rosen[11] and Penrose[12], we consider the novel hypotheis of an *emergent many–body nonlinearity*, whose effective strength depends upon the physical context. Then the successes of microphysics are open to recovery, while the macrophysics is subtly altered. Thus we suppose that the superposition principle is properly a few–particle idealization, a *quantum micro–limit*, as it were.

To interpret the theory we must adopt a scenario where observation is treated as part of physics[2], and not philosophy[7]. The original program of Schrödinger admits a plausible interpretation along these lines[2], and so we revive it. Hereafter, we search after a unified theory of micro and macro physics[5] that is: 1) *General and inclusive*, 2) *Predictive*, and 3) *Self–consistent*. To meet the basic requirement of self–consistency we seek a simple *generic solution* to the problem of dispersion.

5. The physics of dispersion

Let us seek, therefore, a generic physical mechanism for the non–entangling localization of a single scalar massive neutral particle. As a guide we adopt an idea of Rosen[11], and isolate quantum effects in an extra potential which alters classical Hamilton–Jacobi theory. Substituting the Ansatz $\psi = \rho^{1/2} \exp\{iS/\hbar\}$ into

$$i\hbar \frac{\partial \psi}{\partial t} = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + V \right\} \psi, \tag{1}$$

where V is a possibly nonlinear potential (i.e., $V = V[\psi, \psi^*]$), we obtain

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, && (2) \\ m\frac{d\mathbf{v}}{dt} + \nabla(V + Q) &= 0, && (3) \\ \frac{\partial S}{\partial t} + \frac{\nabla S \cdot \nabla S}{2m} + (V + Q) &= 0. && (4) \end{aligned}$$

In this picture, $\mathbf{v} \equiv \nabla S/m$ is viewed as the velocity field of a “fluid”, flowing without loss according to (2), which expresses conservation of the current $\mathbf{j} = \rho \mathbf{v}$.

In the same vein, (3) is the hydrodynamic analog of $\mathbf{F} = m\ddot{\mathbf{x}}$, showing that a gradient force acts on “fluid elements”. It is the sum of V, the Schrödinger potential, and an intrinsically nonlinear *quantum potential*, which reads

$$Q[\psi, \psi^*] \equiv -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} = -\frac{\hbar^2}{4m} \left(\frac{\nabla^2 \rho}{\rho} - \frac{1}{2} \frac{(\nabla \rho)^2}{\rho^2} \right). \tag{5}$$

This is positive for lumped states like gaussians, and increases as they become more localized, but decreases as they become more massive. Only Q is present when V vanishes, and it is a repulsive potential for a localized state. Thus Q isolates the physics of dispersion in the fictitious "dispersion force" $-\nabla Q$, opposed to $-\nabla V$.

Finally, (4) is the usual classical Hamilton–Jacobi equation

$$\frac{\partial S}{\partial t} + H(\mathbf{x}, \nabla S) = 0, \tag{6}$$

modified by the addition of the term Q to H. In this one–particle scenario the quantum potential appears as the prime modification to classical physics[11].

Since Q and V are additive, but Q depends on ρ, a suitable state may have its dispersion cancelled by V. Indeed, an eigenstate which solves

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \tag{7}$$

also satisfies the equilibrium condition $E = V + Q$, whence $\nabla Q = -\nabla V$, showing that the two "gradient forces" are in balance. Thus we might cancel Q, *for a free particle*, using an additional potential $V_{\text{free}}[\psi, \psi^*]$[11]. Recall that Schrödinger used Hamilton–Jacobi theory to find the microscopic equation[2]. Here we reverse the historical argument to constrain a physical dispersion–free macroscopic equation.

6. Gravitation and the physics of free particles

The localizing term we seek must be universal, or one is back again to observers. Since all particles carry mass–energy they are all subject to gravitation. Nor does the formation of gravitational bound states ever lead to screening of the source term via the sum of opposite charges. Thus it is omnipresent. Further, Q is a *repulsive potential* for those states which are well–localized. A generic force of localization must then have an attractive potential. Gravitation is thus, and universally so. If we look now upon Q, it is remarkable that it depends only upon m, and the localization scale of ρ. In gravitation we find a matching source term behaviour so that the two may work in concert, in a due exchange of dominancy.

In order to locate a term in this role we must identify a new physical effect that is excluded from the free–particle wave equation. With an otherwise empty universe the only candidate is gravitational self–interaction.

Recall that for non–gravitational interactions these are included when the field and particle are second–quantized. The classical infinities of a point–like source are banished via covariant cancellation to all orders in a perturbatively renormalizable gauge field theory. All is accounted for, there is no "missing term" due to these. However, although classical gravitation is a gauge theory, the pattern of it differs from the other three forces[21]. The interesting sector of relativistic quantum gravity is inherently non–perturbative, and the standard theory is non–renormalizable, and thus unpredictive[17]. Given the serious and persistent nature of this problem we will explore a nonlinear, and non–perturbative treatment of gravitational self–energy which is *already finite*, and so does not demand second–quantization.

7. The Schrödinger treatment of self–energy

To include gravitational self–energy in a non–perturbative manner, and without recourse to second–quantization, we consider

$$\rho(\mathbf{x},t) = m\psi(\mathbf{x},t)\psi^*(\mathbf{x},t) \tag{8}$$

as the mass density for our particle. To meet the demand that the self–field be $1/r$–like at infinity we adopt the classical Poisson equation

$$\nabla^2\Phi_{\text{grav}}(\mathbf{x}) = 4\pi G m\rho(\mathbf{x},t), \tag{9}$$

as its source, and obtain the *gravitational self–potential*

$$\Phi_{\text{grav}}(\mathbf{x}) = -Gm\int \frac{\psi^*(\tilde{\mathbf{x}},t)\psi(\tilde{\mathbf{x}},t)}{|\mathbf{x}-\tilde{\mathbf{x}}|}\, d^3\tilde{\mathbf{x}}. \tag{10}$$

Coupling (10) back upon the particle we compute

$$E_{\text{grav}} = -\frac{Gm^2}{2}\int\int \frac{\psi^*(\tilde{\mathbf{x}},t)\psi^*(\mathbf{x},t)\psi(\tilde{\mathbf{x}},t)\psi(\mathbf{x},t)}{|\mathbf{x}-\tilde{\mathbf{x}}|}\, d^3\tilde{\mathbf{x}} d^3\mathbf{x}, \tag{11}$$

leading to the "mass renormalization" $m \mapsto m + \delta m$, where $\delta m = E_{\text{grav}}/c^2$. For a wavepacket having linear extension ℓ this reads

$$\frac{\delta m}{m} \approx -\frac{Gm}{\ell c^2},$$

so that a nucelon with $m \approx 10^{-27}$kg, and $\ell \approx 10^{-15}$m, yields $|\delta m/m| \approx 10^{-39}$, decreasing as ℓ increases. Evidently, a direct test of Newtonian quantum gravity demands either a very high–energy accelerator experiment,[b] or a highly sensitive test of the many–body coherent dynamics of very large numbers of atomic–scale particles in close proximity. Suffice to say, no experiment I am familiar with excludes the above from consideration, and so it is important to examine it carefully.

8. The formalism of generalized quantum dynamics

To meet the twin requirements of *generality* and *inclusivity* we now embed the above within the larger mathematical system of generalized quantum dynamics due to Kibble[9], and Weinberg[10]. Introducing the energy functionals

$$H_{\text{kinetic}}[\psi,\psi^*] \;=\; \int \frac{\hbar^2}{2m}\nabla\psi^*(\mathbf{x},t)\cdot\nabla\psi(\mathbf{x},t)\, d^3\mathbf{x}, \tag{12}$$

$$H_{\text{potential}}[\psi,\psi^*] \;=\; \int \psi^*(\mathbf{x},t)V(\mathbf{x},t)\psi(\mathbf{x},t)\, d^3\mathbf{x}, \tag{13}$$

[b] Relativistic quantum gravity is generally associated with Planck scale physics: the Planck length $\ell_{\text{P}} \equiv (\hbar G/c^3)^{1/2} \approx 1.6\times10^{-35}$m; the Planck mass $m_{\text{P}} \equiv (\hbar c/G)^{1/2} \approx 2.2\times10^{-8}$kg; and the Planck time $t_{\text{P}} \equiv (G\hbar/c^5)^{1/2} \approx 5.4\times10^{-44}$s. Our proposal is consistent with this view since $\delta m/m$ is order unity if $m/\ell = m_{\text{P}}/\ell_{\text{P}} = c^2/G$, whence $m\Phi_{\text{grav}} \approx mc^2$. Then a relativistic theory of quantum gravity is essential, and the orthodox viewpoint on Planck scale physics is recovered.

whose sum is $H_{\text{total}}[\psi,\psi^*]$, and recalling the standard Legendre transformation, the corresponding non–relativistic Lagrangian functional is deduced from

$$H_{\text{total}}[\psi,\psi^*] = \int i\hbar\psi^*(\mathbf{x},t)\frac{\partial}{\partial t}\psi(\mathbf{x},t)\, d^3\mathbf{x} - L[\psi,\psi^*]. \tag{14}$$

Application of the dynamical principle of least action

$$\frac{\delta}{\delta\psi^*(\mathbf{x},t)}\int L[\psi,\psi^*]\, dt = 0 \tag{15}$$

then recovers the standard linear Schrödinger equation, where we have defined the functional derivatives via[14]

$$\frac{\delta^n F[\psi(\tilde{\mathbf{x}},\tilde{t})]}{\delta^n\psi(\mathbf{x},t)} \equiv \frac{d^n}{d\lambda^n}F[\psi(\tilde{\mathbf{x}},\tilde{t}) + \lambda\delta(\mathbf{x}-\tilde{\mathbf{x}},t-\tilde{t})]\bigg|_{\lambda=0}, \tag{16}$$

and similarly for the conjugate operation. To obtain the generalization of this system of mathematics to include the case of a *nonlinear* potential V we need only recognize that complex–valued fields separate into two real fields. As such, the mathematics of complex–valued nonlinear dynamics is a special case of the usual real–valued Hamiltonian dynamics familiar in classical studies[16].

Using (14), (15) and the definition (16) the generalization is immediate. Thus we obtain the generalized nonlinear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \frac{\delta H[\psi,\psi^*]}{\delta\psi^*(\mathbf{x},t)}, \tag{17}$$

of Kibble[9], and Weinberg[10], here expressed in the functional formulation of Jones[14]. To meet the requirement of inclusivity we further impose the scaling restriction

$$H[\lambda\psi,\psi^*] = \lambda H[\psi,\psi^*] = H[\psi,\lambda\psi^*], \tag{18}$$

with λ arbitrary, and demand that $H[\psi,\psi^*]$ be real–valued, consistent with its role as an energy. The condition (18) was first introduced by Kibble[9], and Weinberg[10], as a way to meet the previous difficulties of some nonlinear theories in respect of keeping non–interacting systems separable[8]. It further enforces the decomposition

$$H[\psi,\psi^*] = \int\int \psi^*(\tilde{\mathbf{x}},t)\frac{\delta^2 H[\psi,\psi^*]}{\delta\psi^*(\tilde{\mathbf{x}},t)\delta\psi(\mathbf{x},t)}\psi(\mathbf{x},t)\, d^3\tilde{\mathbf{x}}d^3\mathbf{x}, \tag{19}$$

symptomatic of a theory founded in projective geometry, and complex numbers. As shown by Jones[16] the decomposition of H into generalized expectation values allows for the recovery of Hilbert space geometry and the operator structure of the linear theory. These are very special requirements one would expect of a fundamental extension to quantum dynamics. Here the usual linear theory will emerge in the quantum micro–limit of an isolated atomic system, where gravitation can be neglected and the property of linear superposition is a good approximation.

9. The one–particle theory

Applying the foregoing to (11), we form the total energy functional

$$H_{\rm total}[\psi,\psi^*] = \int \frac{\hbar^2}{2m} \nabla\psi^*(\mathbf{x},t)\cdot\nabla\psi(\mathbf{x},t)\, d^3\mathbf{x} - \frac{Gm^2}{2N[\psi,\psi^*]} \int\int \frac{\psi^*(\tilde{\mathbf{x}},t)\psi^*(\mathbf{x},t)\psi(\tilde{\mathbf{x}},t)\psi(\mathbf{x},t)}{|\mathbf{x}-\tilde{\mathbf{x}}|}\, d^3\tilde{\mathbf{x}} d^3\mathbf{x}, \tag{20}$$

here scaled by the *norm functional*

$$N[\psi,\psi^*] \equiv \int \psi^*(\mathbf{x},t)\psi(\mathbf{x},t)\, d^3\mathbf{x}, \tag{21}$$

to respect (18). Applying (17), and using (16), we obtain

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left\{-\frac{\hbar^2}{2m}\nabla^2 - \frac{Gm^2}{N[\psi,\psi^*]}\int \frac{\psi^*(\tilde{\mathbf{x}},t)\psi(\tilde{\mathbf{x}},t)}{|\mathbf{x}-\tilde{\mathbf{x}}|}\, d^3\tilde{\mathbf{x}} - \frac{E_{\rm grav}}{N[\psi,\psi^*]}\right\}\psi(\mathbf{x},t), \tag{22}$$

as the one–particle *gravitational Schrödinger equation.*

10. The many–particle theory

Consistent with the preceding non–entangled treatment of gravitation, and to encompass both bosonic and fermionic degrees of freedom, we consider

$$H_{\rm grav}[\Psi,\Psi^*] \equiv -\frac{1}{N[\Psi,\Psi^*]}\int\int d^{3n}\tilde{\mathbf{x}} d^{3n}\mathbf{x} \left(\frac{1}{2}\sum_{i,j=1}^{n}\frac{Gm_i m_j}{|\mathbf{x}_i-\tilde{\mathbf{x}}_j|}\right) \Psi^*(\tilde{\mathbf{x}}_1,\ldots,\tilde{\mathbf{x}}_n;t)\Psi^*(\mathbf{x}_1,\ldots,\mathbf{x}_n;t)\Psi(\tilde{\mathbf{x}}_1,\ldots,\tilde{\mathbf{x}}_n;t)\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_n;t). \tag{23}$$

Equation (17) generalizes to

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_n;t) = \frac{\delta H_{\rm total}[\Psi,\Psi^*]}{\delta\Psi^*(\mathbf{x}_1,\ldots,\mathbf{x}_n;t)}, \tag{24}$$

and we obtain the equation of motion

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_n;t) = \left\{-\sum_{i=1}^{n}\frac{\hbar^2}{2m_i}\nabla^2_{\mathbf{x}_i} + \sum_{i=1}^{n} m_i\Phi(\mathbf{x}_i) - \frac{E_{\rm gravity}}{N[\Psi,\Psi^*]}\right\}\Psi(\mathbf{x}_1,\ldots,\mathbf{x}_n;t), \tag{25}$$

where $\Phi(\mathbf{x})$ is a Hartree–Fock[22] type many–body potential,

$$\Phi(\mathbf{x}) = -\sum_{j=1}^{n}\frac{Gm_j}{N[\Psi,\Psi^*]}\int \frac{\Psi^*(\tilde{\mathbf{x}}_1,\ldots,\tilde{\mathbf{x}}_n;t)\Psi(\tilde{\mathbf{x}}_1,\ldots,\tilde{\mathbf{x}}_n;t)}{|\mathbf{x}-\tilde{\mathbf{x}}_j|}\, d^{3n}\tilde{\mathbf{x}}, \tag{26}$$

The one–body mass density is the source of Φ, so we interpret Newtonian gravitation as a non–sentient “observer” of this one distinguished macroscopic observable.

11. Stationary states of nonlinear dynamics

Weinberg[10] has employed the Rayleigh–Ritz variational principle

$$\frac{\delta}{\delta\psi^*(\mathbf{x},t)}\left(\frac{H[\psi,\psi^*]}{N[\psi,\psi^*]}\right)=0, \tag{27}$$

to define the canonically invariant[16] *stationarity condition*

$$\frac{\delta H}{\delta\psi^*(\mathbf{x},t)}=\left(\frac{H}{N}\right)\psi(\mathbf{x},t), \tag{28}$$

where $E = H/N$, is the total energy in the stationary state. This is equivalent to making the Ansatz $\psi(\mathbf{x},t) = f(\mathbf{x})e^{-iE_{\rm total}t/\hbar}$, in (17), with $f(\mathbf{x})$ real–valued. Thus Schrödinger's first conception of quantization[2] is here revived as the *nonlinear eigenvalue problem* (28), that which most naturally generalizes: $\hat{H}|\psi\rangle = E|\psi\rangle$[10,16].

12. Cosmical self–consistency of pure gravitation

To (22) we apply (28) to obtain

$$\nabla^2 f(\mathbf{x})+\frac{2m}{\hbar^2}\left[\epsilon-m\Phi_{\rm grav}(\mathbf{x})\right]f(\mathbf{x})=0, \tag{29}$$

as the eigenvalue equation. The solutions to it have been widely studied in the theory of *boson stars*, where (25) is taken as the *Hartree–Fock approximation*[22,23] to the Copenhagen Newtonian quantum gravity[c]. Here we treat it as fundamental.

Spherically symmetric solutions, i.e. S–wave states, have been computed by Bonnazola and Ruffini[24], Thirring[25], Friedberg et al.[26], and Membrado et al.[27] The physical eigenstates must be normalizable, with $\lim_{\rho\to\infty} f^\star(\rho) = 0$. To fix $\epsilon = E_{\rm total} + E_{\rm grav}$, one employs the virial theorem[27], $2E_{\rm kinetic} = -E_{\rm grav}$, to obtain $\epsilon = 3E_{\rm total}$. Using (9) we can eliminate $\Phi_{\rm grav}$ from (29), whence f solves the novel 4–th order nonlinear equation $\nabla^2(\nabla^2 f/f) = (8\pi/a_{\rm g})f^2$, where $a_{\rm g} = \hbar^2/Gm^3$, the *gravitational Bohr radius*, is the key scale parameter. For a nucleon it is around 10^{22}m, and Planck length for a Planck mass elementary particle.

Friedberg et al.[26] have exhibited an homologous family of solutions to (29). All numerical soultions are obtained as rescalings of a universal function. If $f^\star(\rho)$ and $g^\star(\rho) \equiv \Phi^*(\rho) - \epsilon^*$ solve the system $\nabla^2 f^\star = g^\star f^\star$ and $\nabla^2 g^\star = (f^\star)^2$, then

$$f(r) \;=\; \frac{2^{1/2}}{\pi^{1/2}(\gamma_1)^2 a_{\rm g}^{3/2}}f^\star\left(\frac{2r}{\gamma_1 a_{\rm g}}\right) \tag{30}$$

$$\Phi(r) \;=\; \frac{2}{(\gamma_1)^2}\frac{G^2m^4}{\hbar^2}\left\{g^\star\left(\frac{2r}{\gamma_1 a_{\rm g}}\right)+\epsilon^*\right\} \tag{31}$$

[c] The correspondence principle, as formalized by Dirac in his canonical quantization algorithm[6], demands the *entangling* potential $\Phi = -\frac{1}{2}\sum_{i\neq j}^{n} Gm_i m_j/|\mathbf{x}_i - \mathbf{x}_j|$. The Copenhagen theory of Newtonian quantum gravity thus resembles electromagnetism. Further, this route *requires* a perturbatively renormalizable scheme of second–quantized gravity to obtain a finite self–energy. Since H–F energies are generally larger than the "true" Coulomb energies (see e.g., Lieb and Simon Ref. 23) the spectra of both theories differ generically. They are empirically distinguishable.

solve the system $\nabla^2 f = (2m/\hbar^2)gf$ and $\nabla^2 g = 4\pi G m^2 f^2$. Choosing $f^\star(\rho) = 1$, $df^\star/d\rho|_{\rho=0} = 0$, $g^\star(0) = \gamma_0$, $dg^\star/d\rho|_{\rho=0} = 0$, γ_0 is fixed by shooting to meet the requirement $\lim_{\rho\to\infty} f^\star(\rho) = 0$. Physical solutions occur at discrete values of $\gamma_0(n)$, with the quantum number $n = 0, 1, 2, \ldots$ assigned by node counting, and

$$\gamma_1(n) \equiv \int_0^\infty [f_n^\star(\rho)]^2 \rho^2 \, d\rho \tag{32}$$

$$\epsilon^*(n) \equiv \frac{3}{\gamma_1(n)} \int_0^\infty [f_n^\star(\rho)]^2 g_n^\star(\rho) \rho^2 \, d\rho. \tag{33}$$

Universal functions for the ground–state, and self–potential are displayed in Fig. 1 (where: $\gamma_0(0) = -0.919$, $\gamma_1(0) = 3.47$, and $\epsilon^*(0) = -0.979$). Adding more particles changes the localization scale. From (25) we see that k identical collocated bosons have $a_{\rm g}^{(k)} = a_{\rm g}^{(1)}/k$ so that the "size" of an elementary particle depends upon its *context*, e.g. for $k = 10^{23}$ the nucleon size, 10^{23}m, becomes 10^0m.

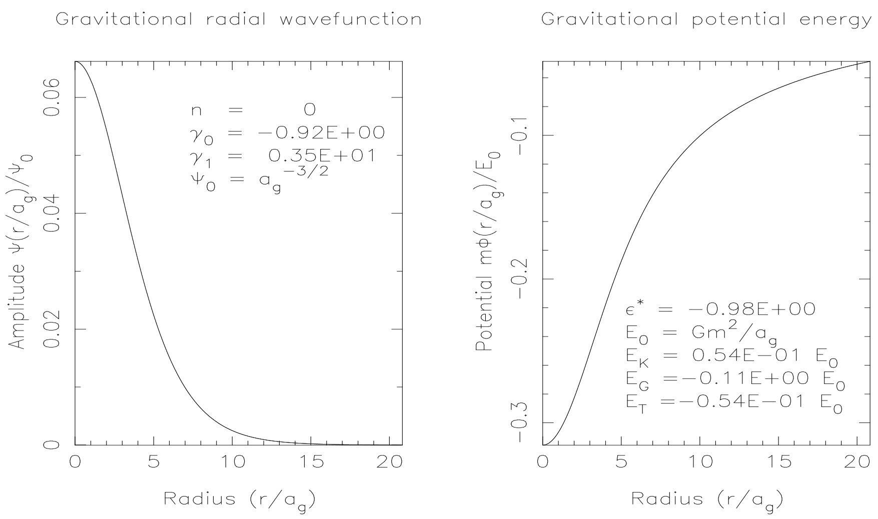

Fig. 1. Universal functions for the gravitational ground state and self–potential. Since no external observer is needed to localize a free–particle the Newton–Schrödinger cosmology is self–consistent. Although gravitation is weak, it does not screen, and so easily dominates macroscopic dispersion.

Of course, these results are not indicative of ordinary matter, where the effects of the Pauli exclusion principle, and the atomic and interatomic binding induced by electromagnetism are decisive[23]. However, in view of the tight localization that is achieved already, we anticipate that the entangling electromagnetic interaction and the dynamic non–entangling gravitational potential will confront one another to introduce elements of intrinsic instability and stochasticity into macrodynamics. With Penrose[12], we suggest that quantum measurement be viewed as a branch of *physics* concerned with the nonlinear *instability* of bulk matter to the inducement of a macroscopic superposition when strongly coupled to a microsystem.

13. Conclusion

The two theories of Newtonian quantum gravity described are each enforced by the correspondence principle, and the vital demand of *interpretational consistency*. They differ quantitatively via the gravitational potential, and qualitatively over entanglement. Work is in progress upon possible empirical signatures, the problem of reconciling waves in configuration space with experience[2,4], and the extension of our predictive nonlinear foil into the relativistic domain of quantum field theory.

Acknowledgements

I thank Bruce McKellar and Mark Thomson for helpful discussions on many–body physics, my colleagues at Queensland, and the Australian Research Council.

References

1. W. Heisenberg, *The Physical Principles of the Quantum Theory* (Dover, New York, 1949); especially Chap. 4, on the Copenhagen interpretation.
2. E. Schrödinger, *Collected Papers on Wave Mechanics* (Blackie & Son, London, 1928).
3. E. Schrödinger, *Ann. d. Phys.* **79** (1926) 734; translation in Ref. 2 pp.45–61.
4. E. Schrödinger, *Naturwiss.* **23** (1935) 807, 823 and 844.
5. E. Schrödinger, *Naturwiss.* **28** (1926) 664; translation appears in Ref. 2 pp. 41–44.
6. P.A.M. Dirac, *The Principles of Quantum Mechanics* 4th edn. (Oxford University Press, London, 1958); Chap. 3 and Chap. 5, see also Ref. 3.
7. N. Bohr, in *Albert Einstein: Philosopher Scientist* ed. P.A. Schlipp (Tudor, New York, 1951); particularly the oft–quoted italicised phrase at the bottom of p.209.
8. I. Biàlynicki–Birula and J. Mycielski, *Ann. Phys. (N.Y.)* **100** (1976) 62.
9. T.W.B. Kibble, *Commun. Math. Phys.* **64** (1978) 73.
10. S. Weinberg, *Ann. Phys. (N.Y.)* **194** (1989) 336; and references therein.
11. N. Rosen, *Found. Phys.* **16** (1986) 687.
12. R. Penrose, in *General Relativity and Gravitation* ed. R.J. Gleiser, C.N. Kozameh and O.M. Moreschi (IOP Publishing, Bristol, 1993).
13. A. Pais, *Rev. Mod. Phys.* **51** (1979) 863.
14. K.R.W. Jones, *Phys. Rev.* **D45** (1992) R2590.
15. K.R.W. Jones, *Phys. Rev.* **A50** (1994) 1062.
16. K.R.W. Jones, *Ann. Phys. (N.Y.)* **233** (1994) 295.
17. C.J. Isham, in *Recent Aspects of Quantum Fields*, H. Mitter and H. Gausterer (eds.), Springer Lecture Notes in Physics **396** (Springer, Berlin, 1992) pp. 123–225.
18. W.H. Zurek, *Phys. Rev.* **D26** (1982) 1862; and references therein.
19. J.J. Bollinger et al., *Phys. Rev. Lett.* **63** (1989) 1031.
20. R. Gähler, A.G. Kelin and A. Zeilinger, *Phys. Rev.* **A23** (1981) 1611.
21. R. Utiyama, *Phys. Rev.* **101** (1956) 1597.
22. A. Kerman and S.E. Koonin, *Ann. Phys. (N.Y.)* **100** (1976) 332.
23. E.H. Lieb and B. Simon, *J. Chem. Phys.* **61** (1974) 735; E.H. Lieb, *Rev. Mod. Phys.* **48** (1976) 553; F.J. Dyson, *J. Math. Phys.* **8** (1967) 1538; and cited articles.
24. R. Ruffini and S. Bonazzola, *Phys. Rev.* **187** (1969) 1767.
25. W. Thirring, *Phys. Lett.* **127B** (1983) 27.
26. R. Friedberg, T.D. Lee and Y. Pang, *Phys. Rev.* **D35** (1987) 3640.
27. M. Membrado, A.F. Pacheco and Y. Sañudo, *Phys. Rev.* **A39** (1989) 4207.