The Importance of Eugenio Beltrami's Hydroelectrodynamics

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Editor's Note

As part of a continuing project of bringing the works of Bernhard Riemann and his collaborators directly before the scientific community, we are happy to present translations of two papers by Eugenio Beltrami that were key in the development of hydrodynamics. We have also translated a short portion of Beltrami's attack upon the electrodynamics of James Clerk Maxwell.

The first paper develops the work of Hermann Helmholtz [translated in IJFE (Helmholtz 1978)] into a generative treatment of vortices. As the note by Dan Wells in this issue (page 59) illustrates in connection with his own work, Beltrami's method has proven critical to the treatment of the development of vortices in plasmas, providing a crucial tool for studying what was otherwise dismissed as turbulence. In his introduction to the translations, Dr. Giuseppe Filipponi develops the connection between the discoveries of Beltrami and the Italian tradition of research in hydrodynamics begun by Leonardo da Vinci.

The role of Leonardo da Vinci as father of the Italian Renaissance and the political activity of Leibniz during the late 17th century clearly demonstrate that scientific advancements are historically born of the political activity required to build republican states committed to scientific, technological, and cultural advancements.

This is exactly the case of the great hydrodynamic school that was developed in Italy during the mid-19th century with the decisive contribution of Bernhard Riemann. Riemann came from Göttingen University to Italy to spend his last years in Pisa and Maggiore Lake. Enrico Betti, Eugenio Beltrami, Felice Groiti, Francesco Brioschi, Luigi Cremona, Masotti, and Carlo Matteucci were the principal representatives of this hydrodynamic school, and they had a decisive role in the political and military struggles that directly led to the formation of the modern Italian state in 1860.

In fact, during the period 1840-1860, as a result of the work of some of these scientists, the Annual Congress of Italian Scientists, which brought together scientists from throughout the Italian peninsula, became one of the centers of the patriotic conspiracy. The Congress was considered so dangerous that two orders were issued in 1850—one by the Austrian governor of Lombardo-Veneto and the other by the Vatican Curia—banning the future participation of scientists from those regions, which were then independent states.

This was also the period during which the Count of Cavour imposed a crash program for the industrialization of the Piedmont region, (then ruled by the King of Savoy) based on steel production and agricultural development. This crash effort provided the essential logistical means for sustaining the military offensives leading into the national unification of Italy.

In spite of World War I and the disastrous fascist regime, the Italian hydrodynamic school, centered in these educational institutions, survived until World War II in the form of the modern aerodynamic school, with Arturo Crocco and Antonio Ferri in Rome, Carlo Ferrari in Turin, and Enrico Pistolesi in Pisa. In 1935, this school founded the Aerodynamic Research Center in Guidonia, near Rome, where Ferri built the most advanced supersonic wind tunnel then found in the world.

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This Italian school was intimately connected with the Prandtl-Busemann School in Germany. After 1945, Antonio Ferri and Adolf Busemann worked in close collaboration at Langley Air Force Base in Virginia.

The Hydroelectrodynamics of Eugenio Beltrami

These historical developments are most important for properly locating the work of Eugenio Beltrami, a student and then colleague of Betti and Riemann, and probably the most brilliant product of the Italian hydrodynamic school. As Betti explicitly reports, the Riemann contribution to the Italian hydrodynamic school was essential for breaking the intellectual chains represented by the mathematical-algebraic approach otherwise prevalent throughout Italian academies and universities.

Under these circumstances, the role of Riemann was essential in forcing Betti, Beltrami, and others to incorporate the method of synthetic geometry into the study of mathematical physics.

Ironically, in the official history of science, Betti and Beltrami are presented as the masters of elasticity theory. What is not reported, however, is that Betti and Beltrami in particular developed this theory for the purpose of demonstrating that the Maxwellian interpretation of electromagnetic phenomena as elastic tension was wrong.

In his paper "On the Mechanical Interpretation of Maxwell's Equations" (see page 51), Beltrami demonstrates that with the Maxwell hypothesis of an elastic ether, it is possible to explain the propagation of electromagnetic potential only in one special case: the potential must result from a geometrical configuration of distribution that can be contracted to a single point.

If the Maxwell hypothesis of an elastic ether has been demonstrated to be wrong, what alternative interpretation do Beltrami, Betti, and Riemann offer in order to explain electromagnetism? This interpretation is presented implicitly in Beltrami's first papers on hydrodynamics presented to the Academy of Science of the Bologna Institute between 1871 and 1874.

These papers, collected under the title "Research on Flow Configuration," describe electromagnetic phenomena as hydrodynamic phenomena. In them, Beltrami elaborated in a three-dimensional field the theory of the potential generated by fluid dynamic vortices and the analogy between the action of these vortices and the electromagnetic action of electrical currents (Figure 1).

This hypothesis was not originally Beltrami's. Riemann had developed the theory of complex functions uniquely to explain hydrodynamic and electrodynamic phenomena, as well as the transmission of heat on two-dimensional surfaces.

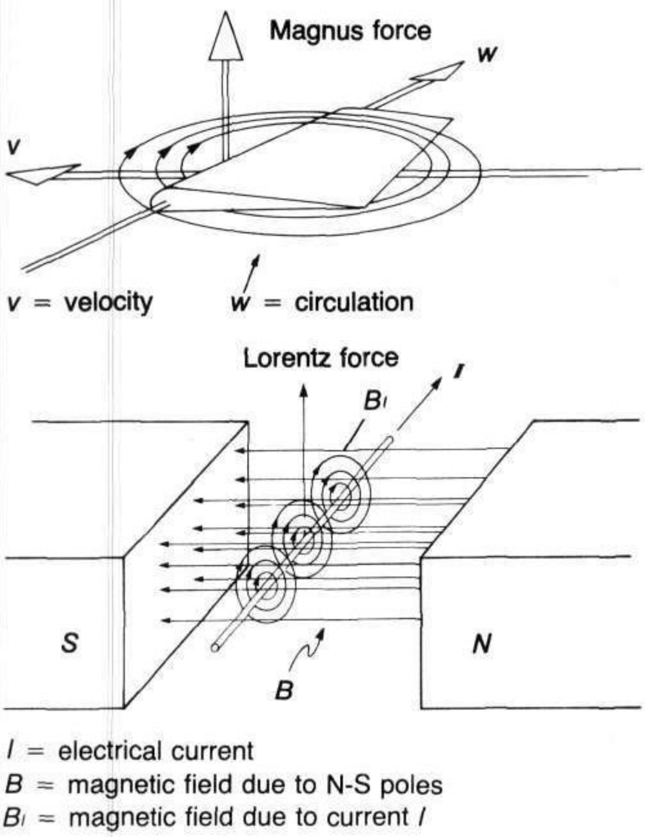

Figure 1. The analogy between the action of fluid dynamic vortices and the electromagnetic action of electrical currents, according to Beltrami.

In 1858, Helmholtz also published a paper on vortices, noting the analogy between the vortices and electromagnetic action. Helmhotz rested his theory on the principle of the Conservation of Energy combined with that of the Conservation of Vorticity. He failed to take into consideration any explanation of the generation of vortices and their evolution.

Beltrami, on the contrary, focused his research on the different possible configurations of fluid vortices in order to discover the principles of the behavior of hydrodynamic and electrodynamic systems. In other words, Beltrami studied hydrodynamic vortices in the same way that the first individuals who developed the techniques of flight found it necessary to study fluid motion both in air and water.

From this standpoint, it can be seen how Beltrami focused his studies on the actual electrodynamic and fluid configurations without concentrating on axiomatic presumptions, such as the conservation laws (as Helmholtz did), and with a particular emphasis on the pathologically unique, extreme cases.

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In his 1889 paper, "Hydrodynamic Considerations," Beltrami presented the hydrodynamic configuration that today bears his name, the Beltrami vortex, in which both the flux and vortex lines are always parallel or antiparallel; that is, when extended to the electrodynamic case, a configuration in which the electric current and magnetic field lines are also always parallel (or antiparallel). In this case, it is found that the "Lorentz" force is zero, and the electric currents do not have to do any work against the magnetic field.

As Beltrami demonstrated, this "force free" configuration is not just some theoretical speculation, but really exists, and precisely describes geometrical configurations that he termed "helicoidal."

The morphology of Beltrami vortices may be seen today in the magnetic geometry of the plasma spheromak configuration and field-reversed pinch. Dr. Winston Bostick of Stevens Institute of Technology in New Jersey discovered the formation of pairs of these force-free filaments in magnetic plasma and plasma pinches as early as 1966 (Figure 2).

Nature then, given the opportunity, demonstrates a pronounced tendency to organize itself according to configurations that correspond to force-free, least-action systems. Research focused on such systems under these conditions can give us the key to comprehending more general physical phenomena, such as the electron, proton, neutron, and so on, if we consider them, not simply as elementary particles, but rather as complex hydrodynamic and electrodynamic systems. The same can also be said for such phenomena as superconductivity and superfluidity in which frictional forces tend toward zero and the efficiency of action is maximal. The Beltrami studies, even if they do not give us a comprehensive theory of hydrodynamic and electrodynamic phenomena, always followed this approach.

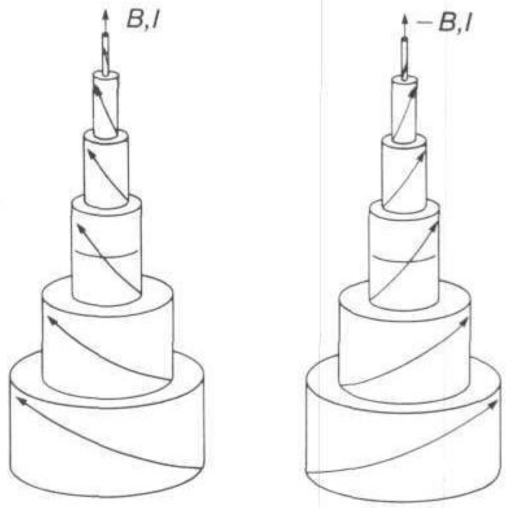

Figure 2. The formation of a pair force-free filaments in magnetic plasma and plasma pinches, according to Dr. Winston Bostick.

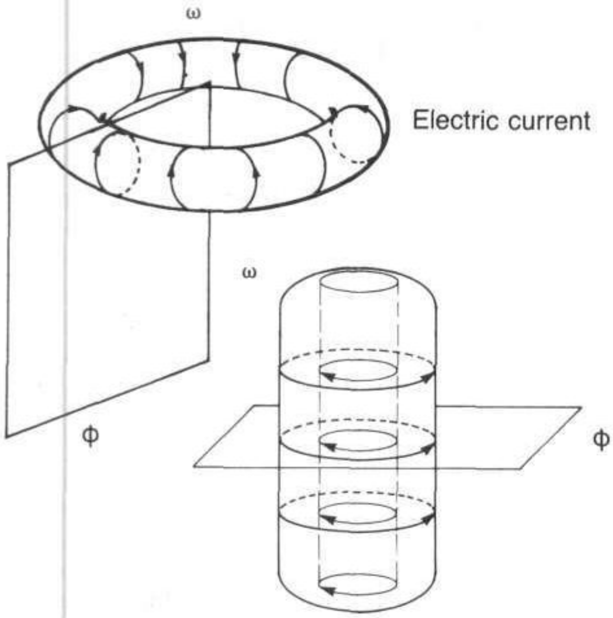

Figure 3. Beltrami's neutral solenoid configuration.

In fact, however, in 1871, in one of Beltrami's first papers, "The Mathematical Theory of Electrodynamic Solenoids," he demonstrated: "Given a surface (ω) shaped as a tube which goes inside itself (torus), which is triply connected, of all the electric currents that can circulate on it, those that exercise the minimum electromagnetic potential on themselves are those that circulate on planes (φ) that are always perpendicular to the (ω) surface (Figure 3).

It is not a coincidence that these configurations are the same configurations found today in leading magnetic fusion experiments like the tokamak, stellarator, and compact tori, which are used to contain the hot fusion plasma.

Beltrami called these configurations neutral solenoids, because the magnetic action is confined inside the surface of solenoids, and outside is zero. The research concerning neutral solenoids is part of Beltrami's work directed toward finding all possible analogies between electrostatic and electrodynamic actions. In the case of neutral solenoids, the analogy is with that of the distribution of electric charges on the surface of a conductor, because the electric action exists only outside the conductor and is zero inside.

The analogies between electrostatic and electrody-

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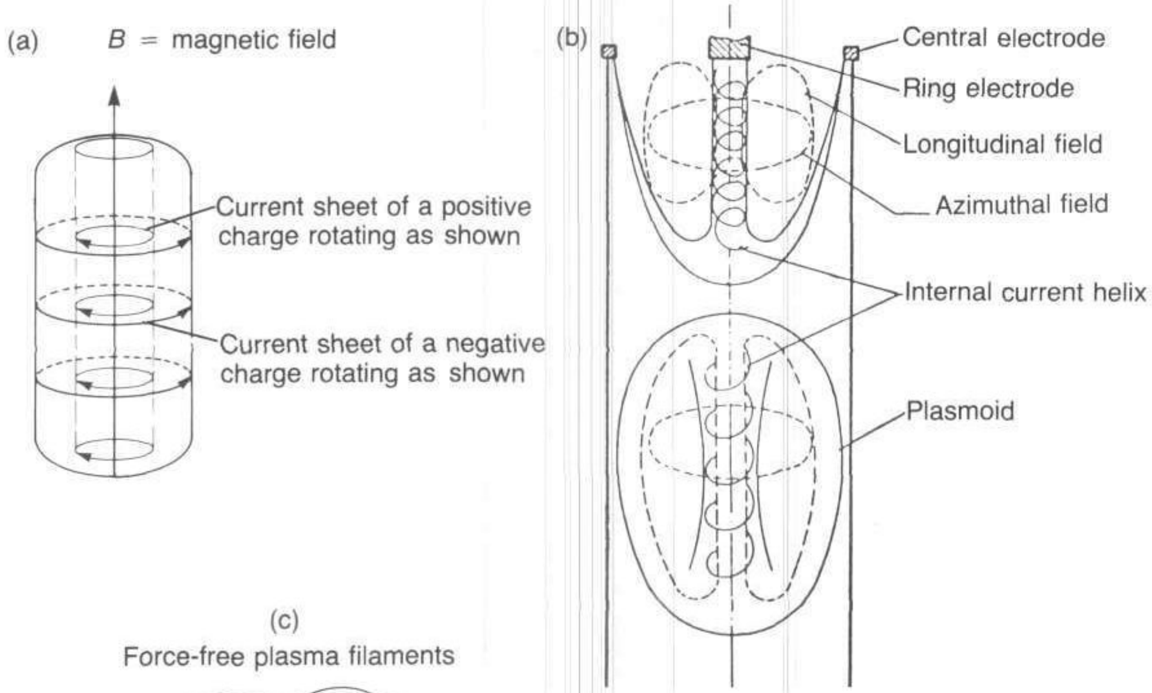
namic phenomena bring us directly to the problem mentioned earlier, that of the possible explanation of the so-called elementary particles as force-free, least-action electrodynamic systems.

The first significant attempt in this direction was carried out by Winston H. Bostick (1985), who describes the electron as a Beltrami electric vortex whose mass is formed by the self-contained electromagnetic energy.

Following the Bostick example, we should be able to develop a model of the neutron according to these conceptions. The idea of developing an electrodynamic model of the neutron is not original. In fact, immediately after the neutron was discovered in 1932, several indirect scattering experiments were done in order to describe the neutron shape. Although the entire science community thought of describing the neutron by the usual Rutherford model—the positive charge in the middle and the negative charge surrounding—the experiments nevertheless showed that there was no "hard" positive charge inside, and that the interior of the neutron was an empty space. Therefore several attempts were made at that time to develop an electrodynamic model of the neutron (Hughes 1959), attempts that were abandoned as the esoteric statistical approach and the quark theory became dominant.

These models were more or less based on the flow of two antiparallel electric currents, one of positive and the other one of negative charges, around a toroidal configuration (Beltrami's tube which goes inside itself). In this electrodynamic configuration, the electrical and magnetic fields are always zero outside the toroidal shape with the exception of the "tube" axis where the magnetic momentum of the two electrical currents are parallel. The neutron does in fact have a magnetic momentum, even if it does not have an electrical charge (Figure 4).

Arturo Crocco, a pioneer of the Italian flight technique, once said that modern aerodynamics had pulled



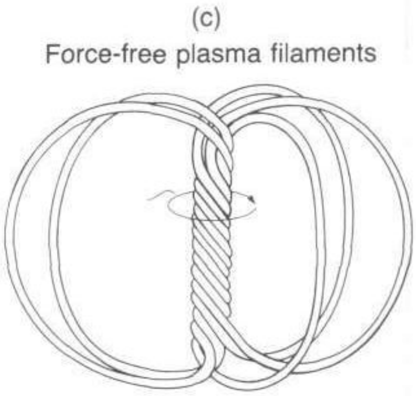


Figure 4. Electrodynamic model of a neutron (a). The two charges, positive and negative, are very close and move in opposite directions. Their magnetic momenta are parallel and the total charge is zero. It will be important to investigate the characteristics of the neutron, in particular, according to a model of a free-force, least-action system. Such a system is shown in (b) and (c), which are examples of plasmoids that self-form in the hot plasma. Such an investigation can give us a deeper knowledge about phenomena like beta decay and nuclear physics in general.

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the vortices of Helmholtz down from Olympus and humanized them. The vortices that Helmholtz described as gods were never born and never die. We can say that plasma physics, so-called particle physics, and other sectors of science can also be brought down from Olympus if we take the same approach to understanding physical phenomena first introduced by Leonardo and refined by the school of Riemann.

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